



U.S. ARMY  
**RDECOM**

TECHNICAL REPORT RDMR-TM-13-01

# AN INVESTIGATION OF ALERTING AND PRIORITIZATION CRITERIA FOR SENSE AND AVOID (SAA)

**Adam Hendrickson**

Technical Management Directorate  
Aviation and Missile Research, Development,  
and Engineering Center

October 2013

Distribution Statement A: Approved for public release;  
distribution is unlimited.



## **DESTRUCTION NOTICE**

**FOR CLASSIFIED DOCUMENTS, FOLLOW THE PROCEDURES IN DoD 5200.22-M, INDUSTRIAL SECURITY MANUAL, SECTION II-19 OR DoD 5200.1-R, INFORMATION SECURITY PROGRAM REGULATION, CHAPTER IX. FOR UNCLASSIFIED, LIMITED DOCUMENTS, DESTROY BY ANY METHOD THAT WILL PREVENT DISCLOSURE OF CONTENTS OR RECONSTRUCTION OF THE DOCUMENT.**

## **DISCLAIMER**

**THE FINDINGS IN THIS REPORT ARE NOT TO BE CONSTRUED AS AN OFFICIAL DEPARTMENT OF THE ARMY POSITION UNLESS SO DESIGNATED BY OTHER AUTHORIZED DOCUMENTS.**

## **TRADE NAMES**

**USE OF TRADE NAMES OR MANUFACTURERS IN THIS REPORT DOES NOT CONSTITUTE AN OFFICIAL ENDORSEMENT OR APPROVAL OF THE USE OF SUCH COMMERCIAL HARDWARE OR SOFTWARE.**

<b>REPORT DOCUMENTATION PAGE</b>			Form Approved OMB No. 074-0188	
Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing this collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503				
<b>1. AGENCY USE ONLY</b>		<b>2. REPORT DATE</b> October 2013	<b>3. REPORT TYPE AND DATES COVERED</b> Final	
<b>4. TITLE AND SUBTITLE</b> An Investigation of Alerting and Prioritization Criteria for Sense and Avoid (SAA)			<b>5. FUNDING NUMBERS</b>	
<b>6. AUTHOR(S)</b>  Adam Hendrickson				
<b>7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)</b> Commander, U.S. Army Research, Development, and Engineering Command ATTN: RDMR-TM Redstone Arsenal, AL 35898-5000			<b>8. PERFORMING ORGANIZATION REPORT NUMBER</b>  TR-RDMR-TM-13-01	
<b>9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)</b>			<b>10. SPONSORING / MONITORING AGENCY REPORT NUMBER</b>	
<b>11. SUPPLEMENTARY NOTES</b>				
<b>12a. DISTRIBUTION / AVAILABILITY STATEMENT</b>  Approved for public release; distribution is unlimited.			<b>12b. DISTRIBUTION CODE</b>  A	
<b>13. ABSTRACT (Maximum 200 Words)</b> <p>Sense and Avoid (SAA) systems require logic that will alert on other aircraft, prioritize amongst other aircraft, and recommend maneuvers. Alerting and prioritizing logic needs to be well integrated for correct prioritization and to reduce nuisance alerts. As a result of the larger time scales involved, the self-separation function will have a greater need to prioritize multiple intruders than collision avoidance. This report explores tau-based criteria when issuing SAA alerts. Deficiencies in current tau-based criteria are detailed. New time estimation criteria are proposed for issuing alerts and prioritizing aircraft. Further work in maturing time-based prioritization criteria is still needed if time estimates are to be used with SAA systems.</p>				
<b>14. SUBJECT TERMS</b> Sense and Avoid (SAA), Unmanned Aircraft, Unmanned Aircraft System (UAS), Traffic and Collision Avoidance System (TCAS)			<b>15. NUMBER OF PAGES</b> 37	
			<b>16. PRICE CODE</b>	
<b>17. SECURITY CLASSIFICATION OF REPORT</b> UNCLASSIFIED	<b>18. SECURITY CLASSIFICATION OF THIS PAGE</b> UNCLASSIFIED	<b>19. SECURITY CLASSIFICATION OF ABSTRACT</b> UNCLASSIFIED	<b>20. LIMITATION OF ABSTRACT</b> SAR	

NSN 7540-01-280-5500

Standard Form 298 (Rev. 2-89)  
Prescribed by ANSI Std. Z39-18  
298-102

## TABLE OF CONTENTS

	<u>Page</u>
<b>I. INTRODUCTION .....</b>	<b>1</b>
<b>II. BACKGROUND INFORMATION ON ALERTING CRITERIA.....</b>	<b>1</b>
<b>III. PHILOSOPHY FOR SOUND SELF-COMPLEMENTARY AND SELF-CONSISTENT LOGIC.....</b>	<b>3</b>
<b>A. Investigation of Tau for Prioritization .....</b>	<b>5</b>
<b>B. Investigation of Non-Maneuvering Trajectories .....</b>	<b>5</b>
<b>C. Investigating Tau Prioritization.....</b>	<b>9</b>
<b>IV. TAU ACCURACY WITH MANEUVERING PATHS LEADING TO MID-AIR COLLISIONS.....</b>	<b>9</b>
<b>V. CHALLENGES USING TAU CRITERIA FOR SENSE AND AVOID .....</b>	<b>12</b>
<b>VI. GROUND RULES FOR MOVING FORWARD .....</b>	<b>12</b>
<b>A. Derivation of Tau-Tau .....</b>	<b>13</b>
<b>B. Prioritization of Encounters .....</b>	<b>17</b>
<b>VII. FURTHER THEORETICAL DEVELOPMENT OF TAU-TAU.....</b>	<b>18</b>
<b>A. Accounting for Turn Rates.....</b>	<b>18</b>
<b>B. Accounting for Slow Closure Rates .....</b>	<b>20</b>
<b>C. Adding a Vertical Component.....</b>	<b>22</b>
<b>D. Adding Right-of-Way Rules .....</b>	<b>24</b>
<b>E. Alerting on Multiple Intruders .....</b>	<b>24</b>
<b>F. Determining Maneuvers .....</b>	<b>26</b>
<b>VIII. OTHER TAU-TAU LOGICS.....</b>	<b>27</b>
<b>IX. ALTERNATIVES TO TIME-BASED LOGIC AND GROUND-BASED SENSE AND AVOID FIELDING PLANS .....</b>	<b>28</b>
<b>X. SUMMARY .....</b>	<b>29</b>
<b>REFERENCES .....</b>	<b>30</b>
<b>LIST OF ACRONYMS, ABBREVIATIONS, AND SYMBOLS .....</b>	<b>31</b>

## LIST OF ILLUSTRATIONS

<b><u>Figure</u></b>	<b><u>Title</u></b>	<b><u>Page</u></b>
1.	Modified Traffic and Resolution Advisory Range Tau Boundaries for TCAS, Sensitivity Level 5 [1] .....	2
2.	Traffic and Resolution Advisory Vertical Tau Boundaries, Sensitivity Level 5 [1] .....	2
3.	Concept of SAA Logic Integration.....	3
4.	Tau Geometry .....	5
5.	30-Second Tau Contour for Parallel Head-On Encounters.....	6
6.	High- and Low-Risk Areas With a Parallel Head-On Engagement Including Standard Rate Turns .....	7
7.	30-Second Tau Contour for Crossing Encounters.....	8
8.	High- and Low-Risk Areas With Crossing Encounter Including Standard Rate Turns.....	8
9.	Plot of Tau Against Real Time for Multiple Co-Altitude Encounters.....	9
10.	Maneuvering Ownship and Non-Maneuvering Intruder .....	10
11.	Maneuvering Aircraft Starting on A-Beam Paths.....	10
12.	Maneuvering Aircraft Starting on Anti-Parallel Routes .....	11
13.	Maneuvers at Larger Tau Values .....	11
14.	Tau Geometry .....	13
15.	Tau-Tau Geometry .....	15
16.	Co-Altitude Parallel Head-On Encounter Comparing Tau and Tau-Tau .....	16
17.	Co-Altitude Crossing Encounter Comparing Tau and Tau-Tau .....	19
18.	Prioritization Encounter Geometries.....	17
19.	Victor Airway Encounter With UA and the Intruder.....	25

## LIST OF TABLES

<b><u>Table</u></b>	<b><u>Title</u></b>	<b><u>Page</u></b>
1.	Quick Look at Tau Suitability for SAA.....	4
2.	Parameters for Figure 18 Geometries .....	17
3.	Example of 15-Knot Horizontal Velocity Constant for Co-Altitude Encounters .....	20
4.	Oscillating Tau-Tau as a Result of Measurement Uncertainty .....	23
5.	Scaling Horizontal Velocity Constant.....	21
6.	Comparing the Effects of Horizontal Velocity Constant on Tau-Tau .....	24
7.	Scaling Factor Applied to $\phi$ for Low Horizontal Velocities.....	26
8.	Alert Combinations Using Inverse and Inverse Square Criteria (Seconds) .....	28
9.	Optical Parameter Combinations That Correlate to Collision Risk .....	27

## I. INTRODUCTION

Sense and Avoid (SAA) systems require logic that will alert on other aircraft, prioritize between aircraft, and recommend maneuvers. Alerting and prioritizing logic needs to be well integrated for correct prioritization and to reduce nuisance alerts. Prioritization of aircraft is especially important for human factors and interaction with the system. As a result of the larger time scales involved, the self-separation function will have a greater need to prioritize multiple intruders than collision avoidance. This report explores tau-based criteria when issuing SAA alerts. Deficiencies in current tau-based criteria are detailed. New time-estimation criteria are proposed for issuing alerts and prioritizing aircraft. Further work in maturing time-based prioritization criteria is still needed if time estimates are to be used with SAA systems.

## II. BACKGROUND INFORMATION ON ALERTING CRITERIA

To set standards to be used in determining requirement thresholds for aircraft avoidance maneuvers, a term called “tau” has been used extensively in the collision avoidance and SAA communities [1]. Tau is commonly defined as being the range between aircraft divided by the range rate between the aircraft:

$$\text{Tau: } \tau = -\frac{r}{\dot{r}} \quad (1)$$

where  $r$  is range, and  $\dot{r}$  is range rate. Sometimes tau is also accompanied by a distance term for issuing alerts. In these cases, a transition in acceptable risk (often triggering an action) is reached if the dynamics between aircraft are within a given tau or distance threshold.

As a result of the deficiencies in the tau definition to account for accelerating aircraft, a modified definition is sometimes used [1, 2], as shown in Equation 2:

$$\text{Distance Modified Tau or “Modified Tau”}: \tau_{DMOD} = -\frac{r - \left(\frac{DMOD^2}{r}\right)}{\dot{r}} \quad (2)$$

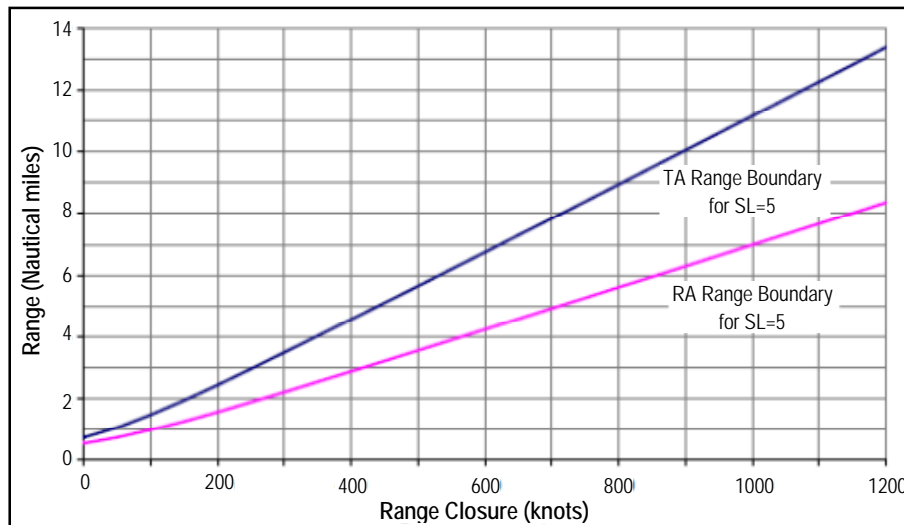
where Distance Modification (DMOD) is a range term that attempts to modify tau to account for scenarios involving slow closing rates where there is a potential risk from accelerating aircraft. A constraint is placed on Modified Tau such that when  $r$  is less than DMOD, Modified Tau = 0 regardless of Equation 2. This condition is required so that an alert is issued when aircraft are in close proximity to each other, even if the range rate is very small. The DMOD tau provides a criterion for alerting with both slow and fast range rates.

Reference 1 provides additional information on the use of the DMOD term and provides DMOD values for Traffic and Collision Avoidance System (TCAS) II. The DMOD term in TCAS changes with sensitivity levels, which in turn correlate to altitude bands.

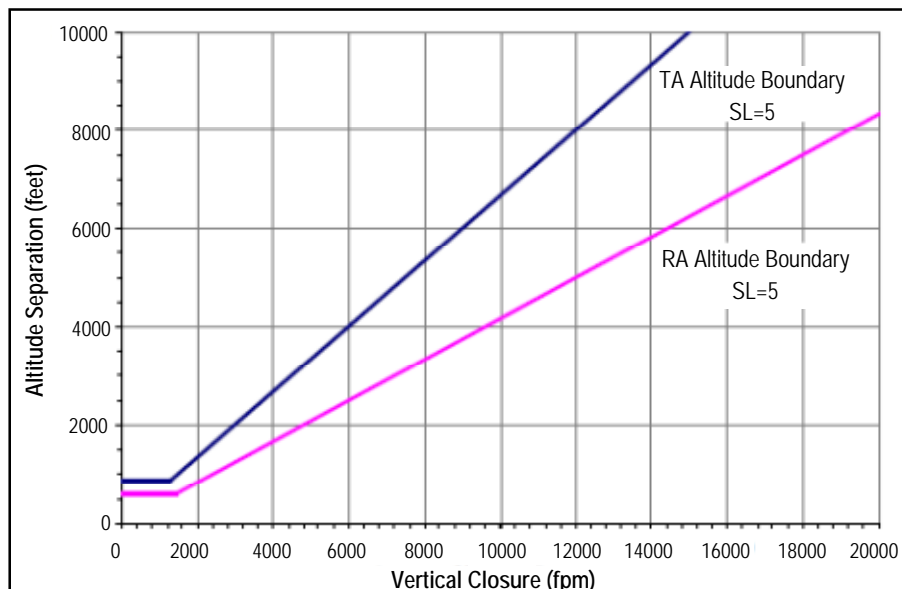
As stated in Reference 1, the TCAS logic uses two different modified tau terms: the range and vertical tau.

“TCAS primarily uses time-to-go to CPA [closest point of approach] rather than distance to determine when a TA or an RA should be issued. The time to CPA is called the range tau and the time to co-altitude is called the vertical tau. Tau is an approximation of the time, in seconds, to CPA or to the aircraft being at the same altitude. The range tau is equal to the slant range (nmi) divided by the closing speed (knots) multiplied by 3600. The vertical tau is equal to the altitude separation (feet) divided by the vertical closing speed of the two aircraft (feet/minute) times 60” [1].

Figure 1 shows an example of traffic and advisory range tau curves that are used in TCAS. Figure 2 shows the range tau curves that are used in the DMOD version.



*Figure 1. Modified Traffic and Resolution Advisory Range Tau Boundaries for TCAS, Sensitivity Level 5 [1]*



*Figure 2. Traffic and Resolution Advisory Vertical Tau Boundaries, Sensitivity Level 5 [1]*



Just as there are range and vertical tau, there is a DMOD value for horizontal distance and an altitude threshold for vertical distance from which traffic and resolution advisories may be issued. The DMOD and altitude threshold values protect against slow lateral and vertical closure encounters. They provide a minimum separation for alerts.

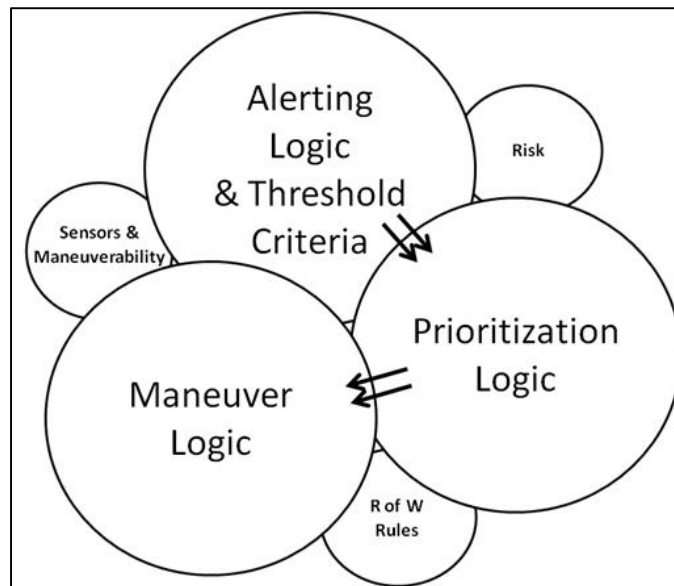
A challenge with using distance modified tau is that it does not allow a means to prioritize between multiple aircraft within the DMOD threshold. It also does not provide a means to prioritize between aircraft that are outside versus inside the DMOD threshold.

### **III. PHILOSOPHY FOR SOUND SELF-COMPLEMENTARY AND SELF-CONSISTENT LOGIC**

The alerting logic should be well integrated with the prioritization logic in a SAA system. Ideally, the two pieces of logic should be based on the same logic structure. For example, if the alerting logic produces an alert at a given tau threshold, the prioritization logic should prioritize intruders based on tau values. If the prioritization logic prioritizes based on some other criteria, then this indicates that there may be a deficiency in the total system such as non-optimal prioritization criteria or nuisance alarms that did not require action, and so forth.

Likewise, the prioritization logic in a SAA system should be well integrated with the maneuver logic. Ideally, the two pieces of logic should be based on the same logic structure. For example, if the alerting logic is tau based, then the maneuver logic should ultimately base maneuver recommendations that are traceable to tau-related criteria.

The concept of self-complementary and self-consistent system logic is simple but profound. The implication is that by defining an alerting logic, constraints are placed on the prioritization and maneuver logic. Otherwise, there is a deficiency in the optimization of the total system. By extension, a trade space exists between the logic pieces, the aircraft maneuverability, and the sensor capability, as shown in Figure 3.



*Figure 3. Concept of SAA Logic Integration*

One of the outcomes of this concept of ideal SAA logic integration is that there may not be single-alerting criteria that are self-consistent and optimized for all SAA systems. Instead, just as there are many classes of Unmanned Aircraft (UA) and classes of sensors, there may be classes of possible SAA algorithms. For illustration purposes, the use of the previous tau criteria is applied to the systems in Table 1.

Table 1. Quick Look at Tau Suitability for SAA

<b>Aircraft Type</b>	<b>Sensor Type</b>	<b>Sensor Outputs</b>	<b>Tau Threshold of 25-Seconds Definitions: (Slant Range/Range Rate) (Altitude/Vertical Rate)</b>
Conventional Manned Aircraft	TCAS	Slant-Range Altitude Slant-Range Rate Vertical Rate	A tau of 25 seconds supports Traffic Advisories from 1,000 to 2,350 feet and Resolution Advisories from 5,000 to 10,000 feet. The system supplements a pilot's ability to "see and avoid."
UAS (Low Maneuverability)	-	-	Likely insufficient maneuverability to support 25-second tau threshold while maintaining acceptable risk ratio in many situations.
UAS (Terminal Environment)	-	-	Tau criteria produce too many alerts to be usable in many scenarios.
UAS Limited Sensor SWAP	EO/IR	Angles Angular rates	Sensor type is not directly compatible with tau definition.
UAS ABSAA	Radar	3-D Position 3-D Rates  (relative to ownship)	Sensor type could conform to above tau definition. Not all of the sensor information is being utilized, suggesting that there may be room for improvement in the alerting logic.
UAS GBSAA	Radar	3-D Position 3-D Rates  (relative to ground radar)	Sensor type could conform to above tau definition. Not all of the sensor information is being utilized, suggesting that there may be room for improvement in the alerting logic.
Human Pilot			Sensor type (pilot eyes) is not compatible with tau definition.

Table 1 shows that there are compatibility road blocks and potential optimization challenges with using the tau definition as the standard alerting logic for all SAA systems. In addition, the tau-alerting construct, which was developed for collision avoidance, may not be appropriate when applied to self-separation criteria or in airspace areas with structured traffic such as the terminal environment.

### A. Investigation of Tau for Prioritization

Testing an alerting logic for prioritization reveals two major characteristics about the logic framework:

1. The quality of alerts:
  - a. Nuisance/unnecessary alerts
  - b. Missed alerts
2. Ability to correctly prioritize between multiple intruders

If an alerting logic cannot correctly prioritize among multiple intruders, it will also likely have deficiencies in the quality of alerts being issued. A good prioritizing logic also makes for a good alerting logic.

### B. Investigation of Non-Maneuvering Trajectories

The tau estimate (range/range rate) allows for a simple formula that can be easily manipulated. Equation 3 is the geometry shown in Figure 4:

$$\tau = -\frac{r}{\dot{r}} = -r / (V_1 \cos(\vartheta_1) - V_2 \cos(\vartheta_2)) \quad (3)$$

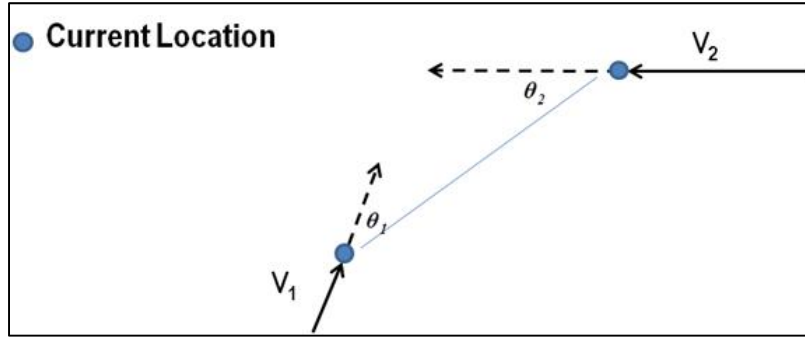
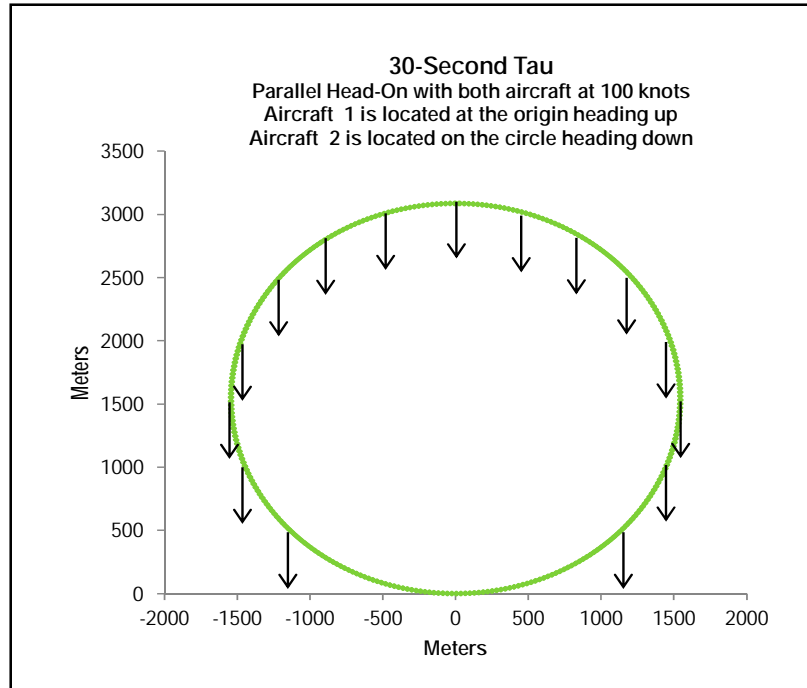


Figure 4. Tau Geometry

If the tau equation is reorganized, it can easily be shown in polar coordinates. The fact that the equation can so easily be arranged into polar coordinates is an important insight and will become evident in Figures 5 through 8 where the contour shapes are circles.

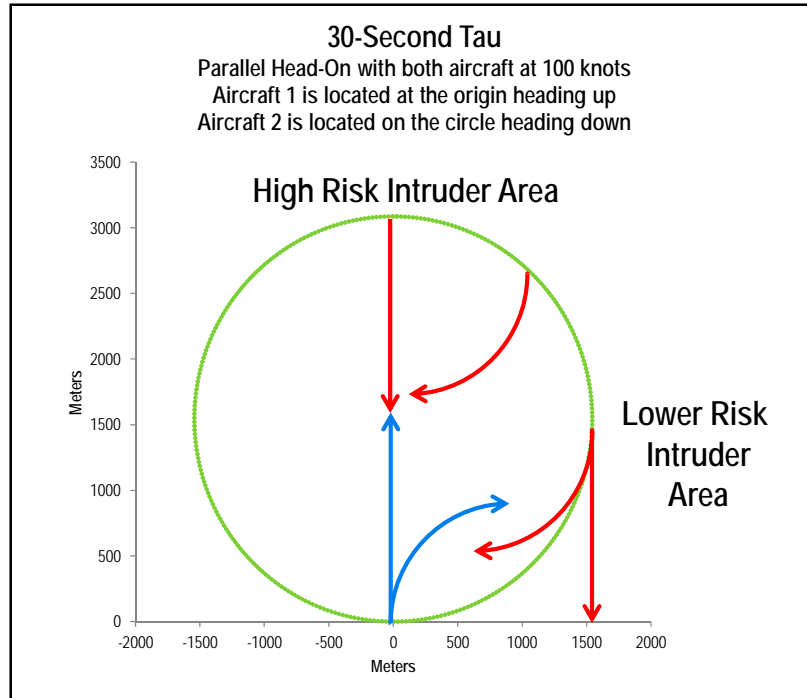
Using the basic tau equation (without DMOD) and plotting constant tau contours reveals interesting insights into the criterion. Tau contours are shown in Figures 5 through 8. Figure 5 shows a constant tau value across the horizontal plane between two co-altitude aircraft in a head-on parallel trajectory engagement. Aircraft 1 is ownship (presumably the UA) and it is located at the origin at the bottom of the circle and heading upwards. Aircraft 2 (the intruder) is located at any location on the contour and heading downwards. Any location of Aircraft 2 placed on the contour produces a tau value of 30 seconds. The shape of the tau contours are the same regardless of a 30- or 60-second tau.



*Figure 5. 30-Second Tau Contour for Parallel Head-On Encounters*

The heading of Aircraft 1 (ownship at the origin) and 2 (anywhere on the contour) are opposite in direction and parallel. If Aircraft 2 is placed at the top of the circle, both aircraft will be on a direct collision trajectory. All other locations on the 30-second tau circle are parallel head-on encounters that do not result in a collision unless maneuvers are executed. Therefore, while any position on the circle has the same tau value, the location at the top of the circle is a high risk location for Aircraft 2 relative to Aircraft 1.

Figure 6 shows the high- and low-risk intruder areas. If Aircraft 2 is located on the side of the circle, it will cross by Aircraft 1 on a parallel trajectory in 15 seconds. If Aircraft 2 immediately begins a standard rate turn, it would not be able to cause a near mid-air collision. Even if Aircraft 1 and 2 immediately begin a standard rate turn towards each other, the aircraft will not collide (or result in a near mid-air collision), as shown in Figure 6. This initial location of Aircraft 2 on the side of the circle is a lower risk location than at the top of the circle even though both locations have the same 30-second tau value. While a tau value of 30 seconds is often associated with a collision avoidance event (depending on altitude), in reality there may be a relatively low probability of a collision actually occurring, depending on the geometry and the speed of the aircraft.



*Figure 6. High- and Low-Risk Areas With a Parallel Head-On Engagement Including Standard Rate Turns*

In the Figure 6 scenario, Aircraft 1 (ownship) is at the origin and enjoys a high degree of safety if Aircraft 2 (Intruder) is in the lower risk intruder area. In fact, if Aircraft 1 continues on its current trajectory, the only way for Aircraft 2, in the lower risk intruder area, to cause a collision is for Aircraft 2 to either do an instantaneous 90-degree turn or a combination of extreme hard turns and accelerations.

The Figure 6 scenario clearly highlights the lower risk intruder areas where nuisance alerts (and poor prioritization) may occur when using a tau-alerting criteria and threshold.

Similar analysis can be performed for a crossing scenario. In Figure 7, Aircraft 1 (ownship) is at the origin heading up. Aircraft 2 is at any location on the circle heading left (only a portion of the circle is shown). The tau criteria produce a 30-second value if Aircraft 2 is anywhere on the circle. If neither aircraft maneuvers, a collision only occurs if Aircraft 2 is located on the circle at 45 degrees from the origin. This is a high risk location on the circle, even though every point on the circle has a tau value of 30 seconds.

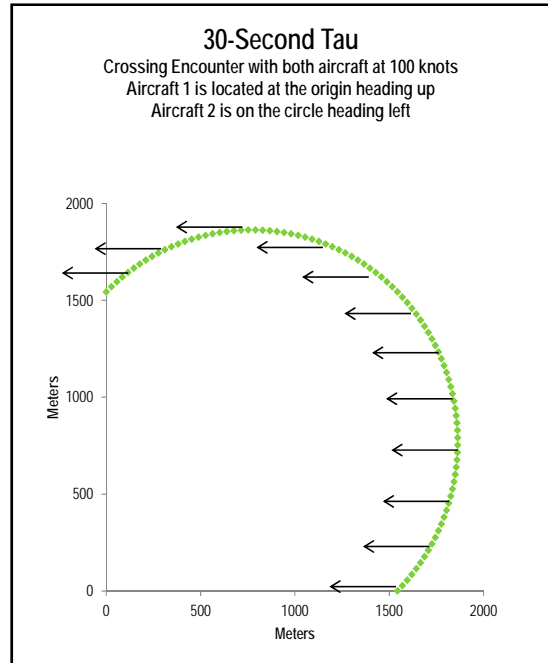


Figure 7. 30-Second Tau Contour for Crossing Encounters

Figure 8 shows the high- and low-risk intruder areas. If Aircraft 2 is located in a lower risk intruder area and immediately begins a standard rate turn towards Aircraft 1, it will not be able to cause a collision without some additional acceleration. The lower risk intruder areas shown in Figure 8 would cause nuisance alerts if collision avoidance alerts were issued at a tau value of 30 seconds.

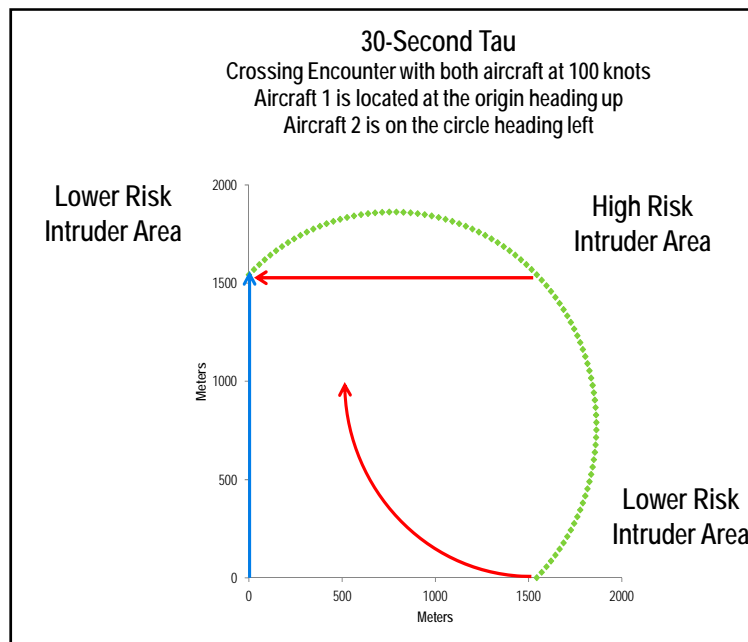


Figure 8. High- and Low-Risk Areas With Crossing Encounter Including Standard Rate Turns

### C. Investigating Tau Prioritization

Figure 9 is an example of how tau changes in real time with three intruder scenarios. Note that the parameters are different than in the previous example. Three encounters were plotted in Figure 9: parallel head-on, crossing, and direct head-on. Figure 9 was set up so that all three intruder aircraft had a point in real time of identical range, range rate, and tau (but not identical position). For encounters that do not result in a collision, tau will diminish until it reaches a minimum and then will continue to grow until it reaches infinity as the range rate approaches zero. In Figure 9, tau gets the prioritization completely wrong on the left portion of the chart. As the encounters evolve past their minima, then tau is able to prioritize correctly; however, by then the UA may already be committed to certain maneuvers. If the intruder has passed the tau minima, then tau is increasing even though the aircraft may still be decreasing in range.

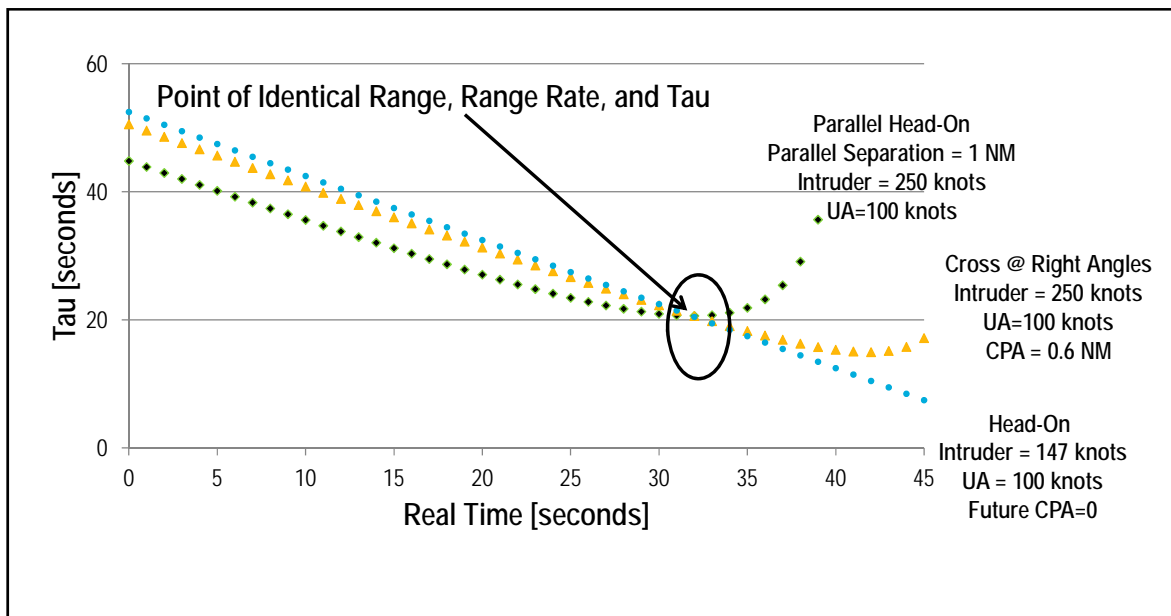


Figure 9. Plot of Tau Against Real Time for Multiple Co-Altitude Encounters

### IV. TAU ACCURACY WITH MANEUVERING PATHS LEADING TO MID-AIR COLLISIONS

The UA flight paths may include loiter patterns, search patterns, traffic patterns, and various turning maneuvers. Similar behavior can be observed, perhaps to a lesser extent, in manned aviation. Figures 10 through 12 test how well tau estimates the time to collision for these scenarios.

Aircraft 1 is in a loiter pattern at a standard rate turn.  
Aircraft 2 is heading straight. Flight paths lead to a collision.

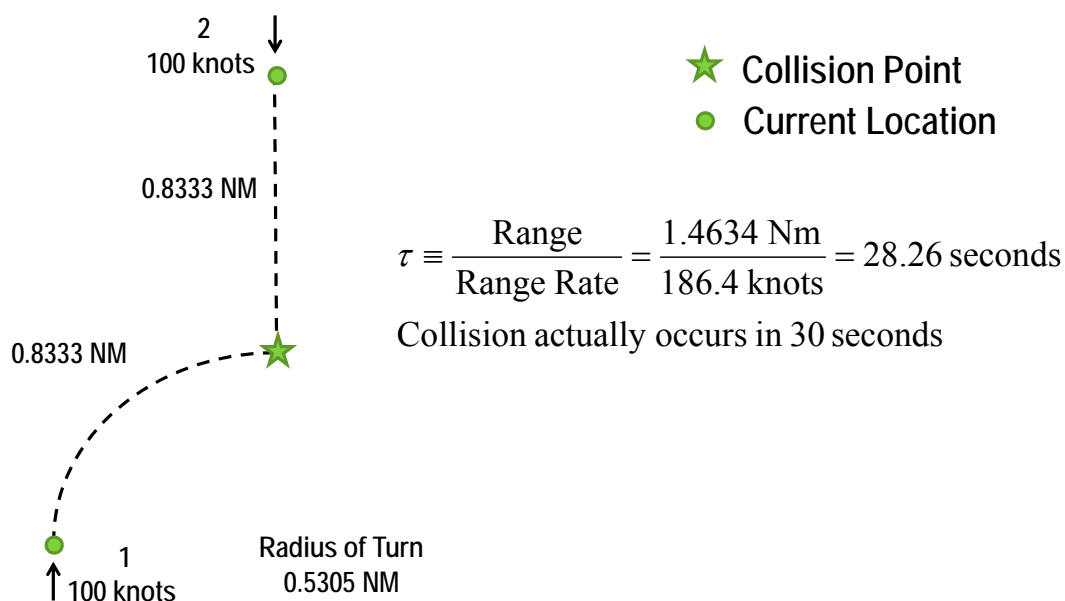


Figure 10. Maneuvering Ownship and Non-Maneuvering Intruder

Both Aircraft 1 & Aircraft 2 are in a standard rate turn.  
Flight paths lead to a collision.

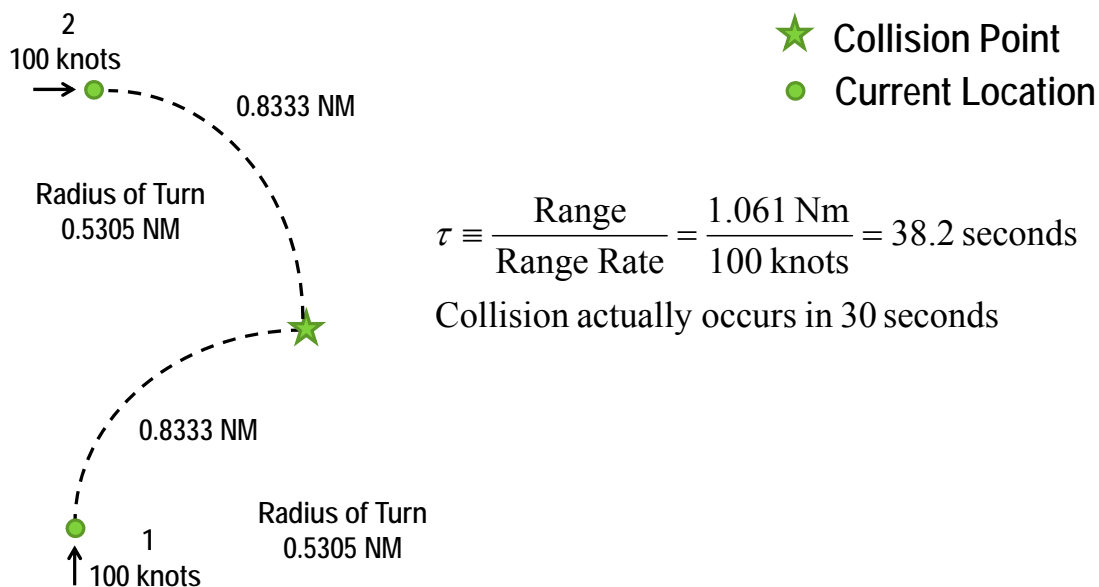


Figure 11. Maneuvering Aircraft Starting on A-Beam Paths



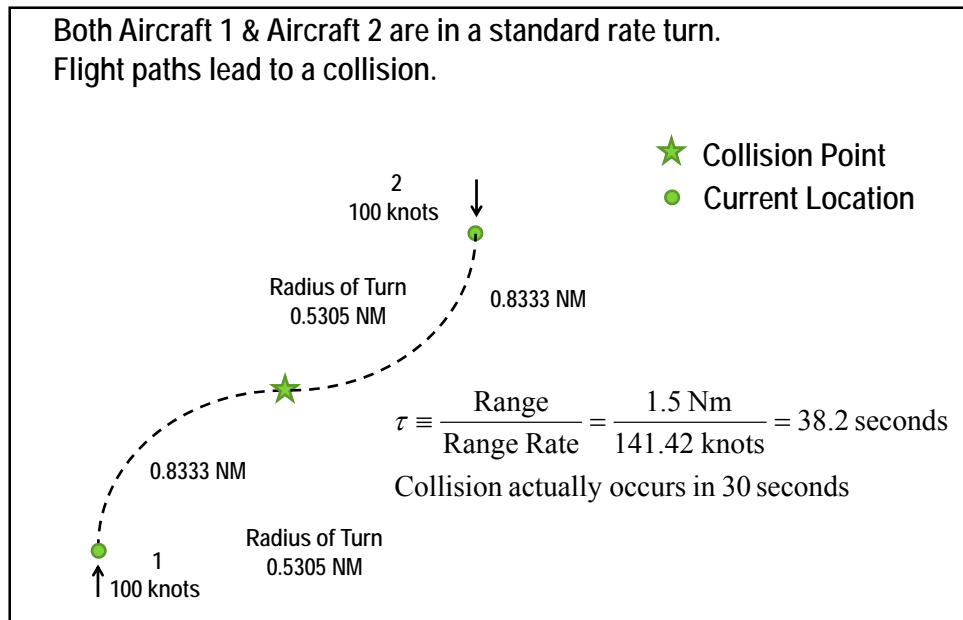


Figure 12. Maneuvering Aircraft Starting on Anti-Parallel Routes

In the collision avoidance case (Figure 10), the tau estimate is not too far off from the actual time to collision. For small standard-rate turns under 90 degrees, tau is a good time estimate for paths leading to a collision as long as only one aircraft is maneuvering.

In Figure 13, the tau estimate has significantly diverged from the actual time to collision. Aircraft turns greater than 90 degrees can cause the tau estimate to quickly degrade in accuracy.

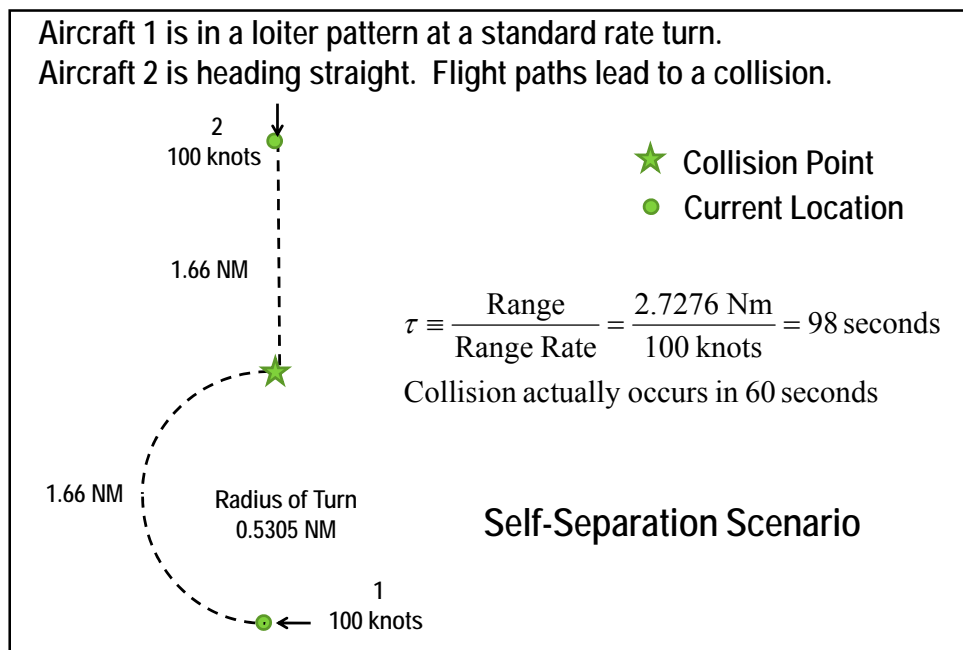


Figure 13. Maneuvers at Larger Tau Values

Figure 13 demonstrates that tau criteria as a time estimate involving maneuvers leading to a collision are not always accurate. The quality of the tau estimate of the time to collision depends on the type of maneuvers. The quality of the tau estimate appears to degrade for larger time scales for maneuvering aircraft. Tau is a sub-optimal estimate of the time to collision and a poor prioritization metric for maneuvering aircraft.

## **V. CHALLENGES USING TAU CRITERIA FOR SENSE AND AVOID**

There are several challenges when using tau for a SAA system. The following are observations taken from the previous examples (and additional investigations of tau not discussed).

- The derivation of tau assumes non-maneuvering aircraft, and all encounters lead to collision. This is a false assumption for the majority of aircraft engagements.
- Tau produces excessive nuisance alerts in the terminal environment to the extent that it may be operationally unusable for SAA in many situations.
- Tau is often overly conservative in issuing alerts for aircraft that are not on a collision path. This overly conservative assumption actually is a detriment to prioritizing between high- and low-risk intruders.
- The accuracy of the tau estimate to collision for maneuvering aircraft degrades at larger time values such as possibly used for self-separation.
- Intruders with slow closing rates which are located within the DMOD parameter have poor prioritization. All of the prioritization logic is held within the tau equation. How would one prioritize between two scenarios when one aircraft is within the DMOD range and another is not?
- The tau estimate does not lead to an integrated self-consistent and self-complementary alerting/prioritization/maneuver logic. The evidence is excessive nuisance/false alarms in situations of low risk which also results in poor prioritization.

## **VI. GROUND RULES FOR MOVING FORWARD**

An entirely new set of equations are proposed that are tunable to a specific SAA system that needs to be distinguished from previous tau definitions. These new class of equations are called tau-tau.

There is a maxim among physicists that says, “What is a minus sign and a factor of two among friends?” The point is not to lose the overall concept of the problem if there is a slight error in the derivation or if the problem can be expressed differently. For convenience the tau-tau equations will be in positive seconds rather than negative seconds. To derive the solutions in positive or negative seconds is a matter of timeline preference. Also depending on how vectors and angles are defined, there may be a sign change in the denominator.

### A. Derivation of Tau-Tau

Many SAA systems will have sensors capable of Three-Dimensional (3-D) position measurements. These could be automatic dependent surveillance-broadcast, airborne radar, ground radar, lidar, and so forth. These sensors, besides being able to measure range, can also measure angles. These measurements improve time estimates in several ways. Before deriving simple tau-tau which is defined as range divided by effective range rate, it is important to understand the derivation of tau. Figure 14 shows the tau geometric formula.

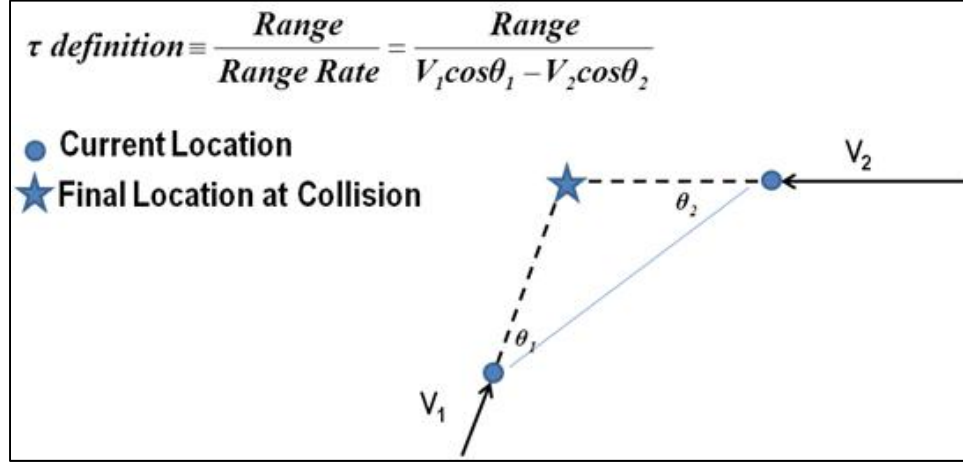


Figure 14. Tau Geometry

The equation of motion that is used for each aircraft is;

$$(x, y)_{final} = (x, y)_{initial} + v(x, y)t \quad (4)$$

where  $v$  is the aircraft velocity, and  $t$  is the time between the initial and final position. Apply this formula to both aircraft, as shown in Equations 5 and 6:

$$\text{Aircraft 1: } (x, y)_{1,final} = (x, y)_{1,initial} + v_1(x, y)t \quad (5)$$

and

$$\text{Aircraft 2: } (x, y)_{2,final} = (x, y)_{2,initial} + v_2(x, y)t \quad (6)$$

Assuming a collision, then  $(x, y)_{1,final} = (x, y)_{2,final}$ . Note that this assumption is not valid for trajectories that do not lead to collision.

$$(x, y)_{1,initial} + v_1(x, y)t = (x, y)_{2,initial} + v_2(x, y)t \quad (7)$$

and

$$t = \frac{(x, y)_{2,initial} - (x, y)_{1,initial}}{v_1(x, y) - v_2(x, y)} = \frac{\text{range}}{\text{range rate}} \quad (8)$$

Sometimes it may be easier to work with the magnitude of the velocity rather than the  $x$  and  $y$  components of the velocity. The cosine of the angle is taken between the velocity vector and the other aircraft and deals directly with the velocity magnitude, as shown in the Equations 9 through 11:

$$v_1(x, y) = V_1 \cos(\theta_1) \quad (9)$$

and

$$v_2(x, y) = V_2 \cos(\theta_2) \quad (10)$$

and

$$t = \frac{\sqrt{(x_{2,initial} - x_{1,initial})^2 + (y_{2,initial} - y_{1,initial})^2}}{V_1 \cos(\theta_1) - V_2 \cos(\theta_2)} = \frac{range}{range\ rate} = \tau \quad (11)$$

To calculate the range rate between aircraft, draw a line between them, as shown in Figure 14. The component of the velocity vector that falls on the line between the aircraft is found by taking the cosine of the angle. This is repeated for each aircraft. However, for a sensor that only measures range, it is not necessary to derive  $\theta_1$  and  $\theta_2$ . The sensor can measure how the range changes with time from an ownship reference frame.

The derivation of tau assumes that both aircraft end up in a final position, which is the collision point. However, this is not the case for the vast majority of aircraft engagements. To derive an estimate of the time to collision (if the aircraft paths do not lead to a collision) is a paradox since the time it takes for two trajectories not on a collision path to collide is infinite.

To include aircraft that are not on a collision trajectory, weight them by how close they are to being on a collision path. This is done by determining the component of the current vector for each aircraft that lies on a collision path. This is similar to the basic math problem of taking a vector and finding the component that lies upon the X-axis. The  $x$  component is found by taking the cosine of the vector. The same principle is applied, but instead of finding the  $x$  component of the vector, the component on a collision path is found. A similar approach was used for finding the range rate between aircraft in the previous tau equation.

This approach scales the velocity vector and legitimizes the attempt to derive a time estimate to collision when aircraft are not on collision trajectories. It allows the computation of when the collision component of Aircraft 1 will be at the same location as the collision component of Aircraft 2 in the derivation of the time estimate.

For a slow aircraft coupled with a fast aircraft, there may not always be a possible collision trajectory that the slow aircraft could cause with a different heading. However, a slow aircraft could still be on a trajectory that increases the risk of collision should the faster aircraft maneuver. Therefore, instead of using the collision vector, use the vector that maximizes the risk of collision. This allows for the inclusion of slower aircraft that may not be able to cause a collision on their own. It is a subtle change, and for many cases, it does not make any difference.

Therefore, tau-tau is defined by the range between aircraft divided by the range rate to maximum collision risk between aircraft.

Tau-tau is better than tau because it estimates the time to collision and creates an effective time estimate that captures aircraft which are not on collision paths by scaling the time estimate based on heading changes needed to cause a collision. Tau-tau is not only a time estimate, but it becomes a means to prioritize between collision and non-collision trajectories. Scenarios with a lower tau-tau value are at higher risk of a collision. Figure 15 shows the geometry and the mathematics of the concept. Equation 12 is the mathematical equation, and Equation 13 is the mathematical concept.

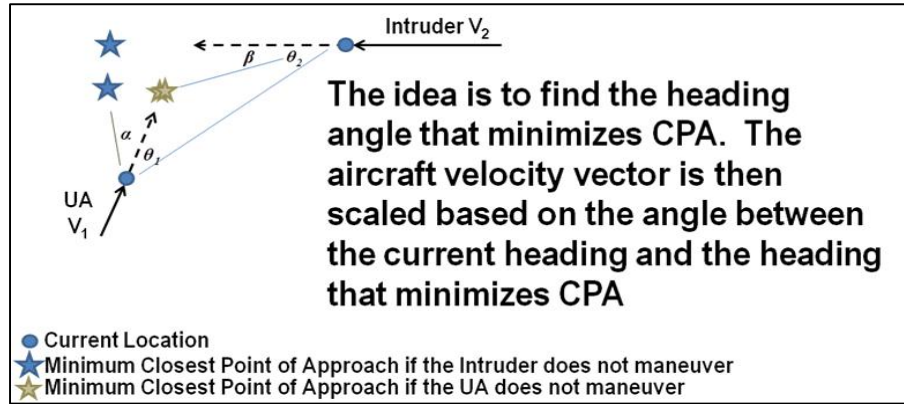


Figure 15. Tau-Tau Geometry

$$\text{tau tau} = \frac{\sqrt{(x_{2,\text{initial}} - x_{1,\text{initial}})^2 + (y_{2,\text{initial}} - y_{1,\text{initial}})^2}}{V_1 \cos(\theta_1) \cos(\alpha) - V_2 \cos(\theta_2) \cos(\beta)} \quad (12)$$

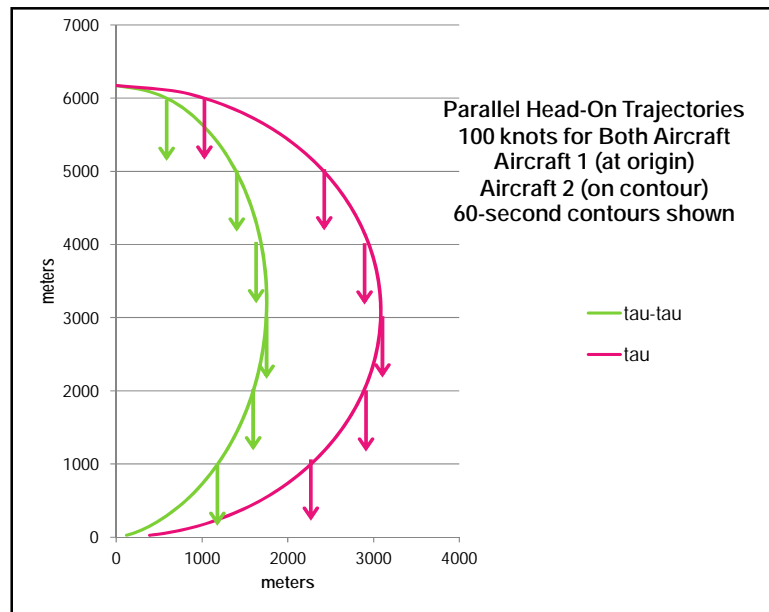
and

$$= \frac{\text{range}}{\text{"range rate to maximum collision risk"}} \quad (13)$$

where,  $\alpha$  and  $\beta$  are the angles between current headings and the headings that minimize the Closest Point of Approach (CPA). The  $\alpha$  is found by projecting  $V_2$  and determining the heading of  $V_1$  (using the current speed) that would minimize the CPA (and maximize collision risk). Likewise,  $\beta$  is found by projecting  $V_1$  and determining the heading of  $V_2$  (using the current speed) that would minimize the CPA. For aircraft already on a collision trajectory,  $\alpha$  and  $\beta$  equal zero and the tau-tau equation reduces to the tau equation providing an accurate time estimate to collision.

Figure 16 compares the tau and tau-tau logic for a parallel head-on encounter. Both aircraft are traveling at 100 knots but in opposite directions. Aircraft 2 (the intruder) will have a 60-second value if it is located on one of the contours. The previous 60-second tau-tau contour is a contour of near equal prioritization. If Aircraft 2 is directly at the top of the contour, no turning is required to cause a collision. Moving down the tau-tau contour, the amount of turning required to cause a collision is effectively traded for the amount of time to collision if either aircraft should suddenly maneuver. Moving down the tau-tau contour, more aggressive

maneuvers are required to cause a collision. Tau-Tau reduces nuisance alarms by reducing the low risk areas on the contour.



*Figure 16. Co-Altitude Parallel Head-On Encounter Comparing Tau and Tau-Tau*

Figure 17 compares tau and tau-tau for crossing encounters. Aircraft 1 (ownship) is located at the origin and heading up along the Y-axis. Aircraft 2 (intruder) is located anywhere on the 30-second contour and is traveling parallel to the X-axis towards the left. Both aircraft are traveling at 100 knots. If both aircraft maintain their trajectories, the only location of the intruder aircraft that will result in a collision is at 45 degrees from the origin. The tau-tau contour avoids the low-risk areas where tau intersects with the X- or Y-axis. The tau-tau contour is a better match to the actual risk of collision since it estimates the range rate not to the other aircraft but to the other aircraft that minimizes CPA.

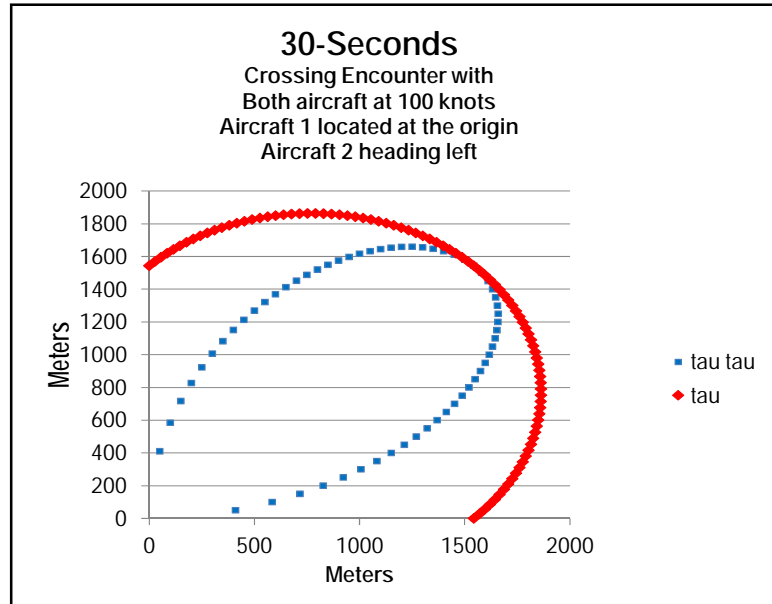


Figure 17. Co-Altitude Crossing Encounter Comparing Tau and Tau-Tau

## B. Prioritization of Encounters

The use of tau-tau allows for prioritization of encounters. Compare the following three co-altitude encounters that are shown in Figure 18 and Table 2. Note the difference between the tau and the tau-tau values for prioritization purposes.

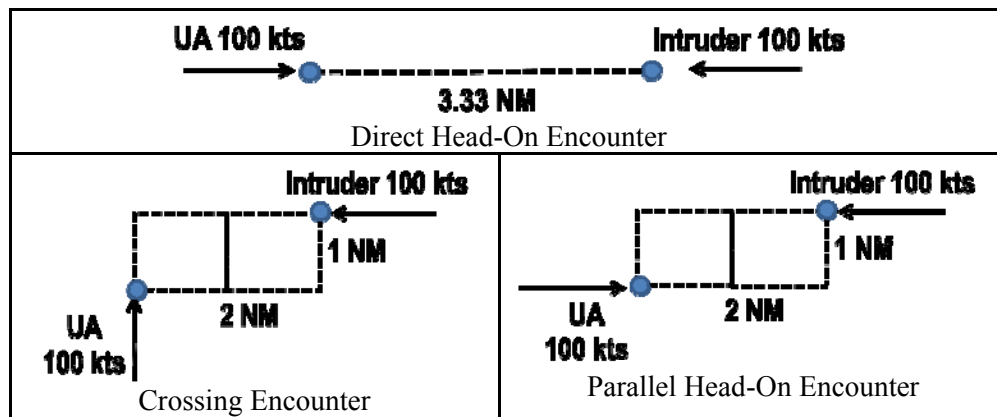


Figure 18. Prioritization Encounter Geometries

Table 2. Parameters for Figure 18 Geometries

Encounter Type	Direct Head-On	Crossing	Parallel Head-On
Ownship Speed	100 knots	100 knots	100 knots
Ownship Heading	East	North	East
Angle to Intruder	$0^0$	$63.435^0$	$-26.565^0$
alpha	$0^0$	$36.87^0$	$-53.13^0$

Intruder Speed	100 knots	100 knots	100 knots
Intruder Heading	West	West	West
Angle to Ownship	0 <sup>0</sup>	-26.565 <sup>0</sup>	-26.565 <sup>0</sup>
Beta	0 <sup>0</sup>	Angle that minimizes CPA= 31.08 <sup>0</sup>	-53.13 <sup>0</sup>
Range	3.33 NM	2.236 NM	2.236 NM
Range Rate	200 knots	134.164 knots	178.885 knots
Tau	60 seconds	60 seconds	45 seconds
Time to CPA	60 seconds	54 seconds	36 seconds
CPA	Collision	0.707 NM	1 NM
Effective Range Rate	200 knots	112.38 knots	107.33 knots
Tau-Tau	60 seconds	71.6 seconds	75 seconds
Correct Prioritization Order 1= Highest Priority 3= Lowest Priority	1	2	3

## VII. FURTHER THEORETICAL DEVELOPMENT OF TAU-TAU

The co-altitude version of the tau-tau equation for non-maneuvering aircraft has been introduced and the derivation explained. Maneuvering aircraft, the inclusion of vertical rates, and a proposal to handle slowly converging aircraft are still under investigation; however, other theoretical approaches may exist.

For completeness in showing how the tau-tau equations could be extended further, right-of-way rules and maneuvers are explored. However, these last two extensions are not meant to be used but to demonstrate the possibility of a well-integrated, self-complementary, and self-consistent logic.

For prioritizing between multiple aircraft, a single time estimate is desired. If instead there are horizontal and vertical time-based alerting equations, the question of correct prioritization becomes complicated very quickly for a multitude of engagements. Consider the case where one aircraft descends in front of another but is not on a collision path. A vertical equation may give one time estimate and a horizontal equation may give another estimate. Now, add a second aircraft that may be a little further out but is on a collision trajectory. The need for a single equation that prioritizes in all three dimensions becomes apparent.

### A. Accounting for Turn Rates

The basic equation framework for tau-tau assumes that aircraft are not currently turning but could turn in the future. This can be problematic for prioritization of aircraft that are currently in a turn. Aircraft motion may include holding patterns near an airport, UA loiter patterns, and so forth. By returning to the equations of motion, equations can be amended to include turn rates. Many SAA systems will have sensors capable of measuring turn rate. A



Careful use of polar coordinates seems to be an ideal coordinate frame to demonstrate the concept.

$$\text{Equation of Motion in Polar Coordinates: } r_f \theta_f = r_i \theta_i + \left( r_i \frac{\partial \theta}{\partial t} + \theta_i \frac{\partial r}{\partial t} \right) t + \frac{1}{2} \left( r_i \frac{\partial^2 \theta}{\partial t^2} + \theta_i \frac{\partial^2 r}{\partial t^2} \right) t^2 \quad (14)$$

Ignore the accelerating range and theta terms, which could be included in other derivations to account for cases where there are low range rates. By applying the equation of motion to two aircraft and setting their final position equal to one another to cause a collision, the time to collision can be found.

$$\tau = \frac{r_{i,1} \theta_{i,1} - r_{i,2} \theta_{i,2}}{\left( \theta_{i,2} \frac{\partial r_2}{\partial t} - \theta_{i,1} \frac{\partial r_1}{\partial t} \right) + \left( r_{i,2} \frac{\partial \theta_2}{\partial t} - r_{i,1} \frac{\partial \theta_1}{\partial t} \right)} \quad (15)$$

and

$$\tau = \frac{r_{i,1} \theta_{i,1} - r_{i,2} \theta_{i,2}}{\left( \theta_{i,2} \frac{\partial r_2}{\partial t} - \theta_{i,1} \frac{\partial r_1}{\partial t} \right) + (r_{i,2} \omega_2 - r_{i,1} \omega_1)} \quad (16)$$

By carefully selecting the correct reference frame, the tau equation that includes aircraft which are currently in a turn is the following:

$$\tau = \frac{\text{range}}{(\text{range rate}) + (\text{tangential rate})} \quad (17)$$

By applying similar steps to the tau-tau equation, the new equation that includes both aircraft which are not currently in a turn as well as those which are is the following:

$$\text{tau tau} = \frac{\sqrt{(x_{2,initial} - x_{1,initial})^2 + (y_{2,initial} - y_{1,initial})^2}}{(V_1 \cos(\theta_1) + r_1 \omega_1) \cos(\alpha) - (V_2 \cos(\theta_2) + r_2 \omega_2) \cos(\beta)} \quad (18)$$

where  $r$  is the radius of the turn and  $\omega$  is the turn rate or rotational speed. Note that when neither aircraft is turning, this form of the tau-tau equation reduces to the previous basic tau-tau equation.

For aircraft that are turning, it is unknown whether they will continue turning or end their turn and proceed on a straight course. The risk of exiting the turn is especially evident in cases when aircraft are not in traffic or loiter patterns; therefore, determine the tau-tau result for both cases (continue the turn or immediately exit the turn). The tau-tau result with the highest priority will be the one used by the system for prioritization against other intruders and issuing alerts.

There is one more adjustment that needs to be made. The  $V_1 \cos(\theta_1)$  term has been scaled by  $\cos(\alpha)$  where  $\alpha$  is the angle between the current heading and the heading required to minimize CPA. A similar scaling is also done for  $r_1 \omega_1$  with one minor exception. For basic encounters, the center of rotation to determine  $\alpha$  is at the aircraft's current location. For turning encounters, the center of rotation is the center of the turn radius where  $r_1 = 0$ .

## B. Accounting for Slow Closure Rates

To account for slow closure rates, examples may include a slow overtake or slowly converging trajectories, where aircraft may become close even though the time to impact may be larger than some alerting threshold. If there is a zero closing rate between aircraft, it is assumed that the risk of a collision increases as the distance between aircraft is reduced. To issue alerts on other aircraft, one approach could insert a DMOD term which has been previously done for tau. However, this destroys the ability for tau to prioritize between aircraft within the DMOD threshold. It also removes the ability to prioritize between aircraft outside versus inside the DMOD threshold.

One approach is to return to the equations of motion and try to include the acceleration terms in the derivation of the time estimate. This approach gets complicated very quickly because acceleration terms are not constant. Another approach is for the logic designer to add a horizontal velocity constant ( $V_H$ ) to the effective rates. For scenarios where there is a zero closing rate, the  $V_H$  term allows distances between aircraft to be directly converted to seconds. This horizontal velocity constant is chosen by the logic designer based on the desired behavior of a specific SAA system. Aircraft in a horizontal flight formation may have a different  $V_H$  term than aircraft not in a formation. Similarly, aircraft in the terminal environment or aircraft on parallel approaches may have a different  $V_H$  term than while they are in transit. Sensor accuracy may also play a part in determining the correct  $V_H$  value. For low closure rates, the impact of the uncertainty in velocity measurements and heading may also be greater, and these effects must be considered. Equation 19 is the tau-tau equation with the horizontal velocity constant.

$$\tau\tau = \frac{\text{range}}{V_H + \text{effective rates}} \quad (19)$$

$$\tau\tau = \frac{\sqrt{(x_{2,initial} - x_{1,initial})^2 + (y_{2,initial} - y_{1,initial})^2}}{V_H + (V_1 \cos(\theta_1) + r_1 \omega_1) \cos(\alpha) - (V_2 \cos(\theta_2) + r_2 \omega_2) \cos(\beta)}$$

Table 3 demonstrates how the addition of the  $V_H$  term that is equal to 15 knots affects tau-tau.

Table 3. Example of 15-Knot Horizontal Velocity Constant for Co-Altitude Encounters

Range (NM)	$V_H$ (Knots)	Effective Rates (Knots)	Tau-Tau Without $V_H$ (Seconds)	Tau-Tau With $V_H$ (Seconds)	Change in Tau-Tau (%)
0.25	15	0	$\infty$	60	
0.50	15	0	$\infty$	120	
1	15	0	$\infty$	240	
2	15	100	72	62.61	-13
2	15	200	36	33.49	-6.97
2	15	300	24	22.86	-4.75

The  $V_H$  term produces more conservative alerting criteria and results in a slightly earlier alert. The  $V_H$  term also helps mitigate tau-tau oscillations because of the uncertainty in sensor measurements of aircraft velocity. This becomes extremely important when prioritizing between various intruders. Table 4 shows an effective rate that oscillates +/- 10 knots as a result of measurement uncertainty.  $V_H$  dampens the oscillation caused by measurement uncertainty especially at slow effective rates.

Table 4. Oscillating Tau-Tau as a Result of Measurement Uncertainty

Range (NM)	$V_H$ (Knots)	Actual Effective Rates (Knots)	+/- 10 Knot Measurement Uncertainty in Effective Rates (Knots)	Tau-Tau Oscillations Without $V_H$ (Seconds)	Tau-Tau Oscillations With $V_H$ (Seconds)
0.25	15	10	-10	$\infty$	60
			0	90	36
			+10	45	25.7
1	15	100	-10	40	34.3
			0	36	31.3
			+10	32.7	28.8
2	15	200	-10	37.9	35.1
			0	36	33.5
			+10	34.3	32

As the effective rates increase, the effect of the  $V_H$  term on the tau-tau estimate decreases. If horizontal measurement accuracy is extremely good or there is a desire for the effect of the  $V_H$  term at larger velocities to become insignificant more quickly, then an exponential term can be applied to  $V_H$ . One way to do this would be to exponentially scale  $V_H$  based on the horizontal effective rate as follows:

$$V_H e^{-|k_H * Effective Rate|} \quad (20)$$

where  $k$  is a tunable scaling term that the logic designer can use to tune the system based on sensor accuracy and thresholds. Table 5 shows the effect of scaling in the  $V_H$  term, and Table 6 shows the impact on tau-tau.

Table 5. Scaling Horizontal Velocity Constant

Effective Rate (Knots)	Scaling Factor, $k$	Horizontal Velocity Constant, $V_H$ (Knots)	Scaled $V_H$ , $V_H e^{- k_H * Effective Rate }$ (Knots)
0	1/40	15	15
10	1/40	15	11.68
20	1/40	15	9.098
40	1/40	15	5.518
60	1/40	15	3.347

80	1/40	15	2.03
100	1/40	15	1.23
200	1/40	15	0.101
300	1/40	15	0.008
Note: k is a scaling parameter.			

Table 6. Comparing the Effects of Horizontal Velocity Constant on Tau-Tau

Range (NM)	Actual Effective Rates (Knots)	$V_H$ (Knots)	$V_H e^{- k_H * \text{Effective Rate} }$ Where $k_H=1/40$ (Knots)	Tau-Tau (Seconds)	Tau-Tau With $V_H$ (Seconds)	Tau-Tau With Scaled $V_H$ Where $k_H=1/40$ (Seconds)
0.25	0	15	15	$\infty$	60	60
0.25	10	15	11.68	90	36	41.5
0.25	20	15	9.098	45	25.7	30.93
1	100	15	1.23	36	31.3	35.56
2	200	15	0.101	36	33.48	35.98
3	300	15	0.008	36	34.28	35.999

### C. Adding a Vertical Component

For alerting and prioritization purposes, it is desirable to have a logic that includes vertical rates. Alerting in the vertical plane is different than alerting in the lateral plane because of an asymmetry that exists as a result of gravity. For example, it is common for an aircraft to perform a 360-degree turn in the horizontal plane but not in the vertical plane. Aircraft behavior in the lateral plane is different than in the vertical plane. This is why a vertical tau was developed. Having both a lateral tau and a vertical tau may work for alerting, but there are additional complexities for prioritization. Having a single equation simplifies the prioritization process.

Use of a single equation that includes both horizontal and vertical components can be advantageous. After testing the behavior of several different types of equations, the following tau-tau equation was proposed:

$$\tau \tau = \frac{(1+|\varphi_1|+|\varphi_2|)(3D \text{ range})}{\text{vertical rates} + \text{effective 2D range rates} + \text{effective 2D tangential rates}} \quad (21)$$

$$\tau \tau = \frac{(1+|\varphi_1|+|\varphi_2|)(3D \text{ range})}{V_{1,z} + (V_1 \cos(\theta_1) + r_1 \omega_1) \cos(\alpha) - V_{2,z} - (V_2 \cos(\theta_2) + r_2 \omega_2) \cos(\beta)}$$

where  $V_1$  and  $V_2$  are the magnitude of the straight line velocity in the horizontal dimension,  $V_{1,z}$  and  $V_{2,z}$  are the vertical velocities,  $r\omega$  is the horizontal tangential rate, and  $\varphi$  is a term like  $\alpha$  and  $\beta$  except that it is the vertical dimension while  $\alpha$  and  $\beta$  remain in the horizontal dimensions. The  $\varphi$  is the angle between the current heading and the heading in the vertical axis required to

minimize CPA. The  $r\omega$  remains in the horizontal plane because aircraft rarely perform sustained vertical turns or loop-the-loops.

Figure 19 is an example where both alpha and beta equal zero. This is a parallel head-on encounter that is separated in the vertical plane as might be observed during operations in a victor airway. All aircraft are at level flight.

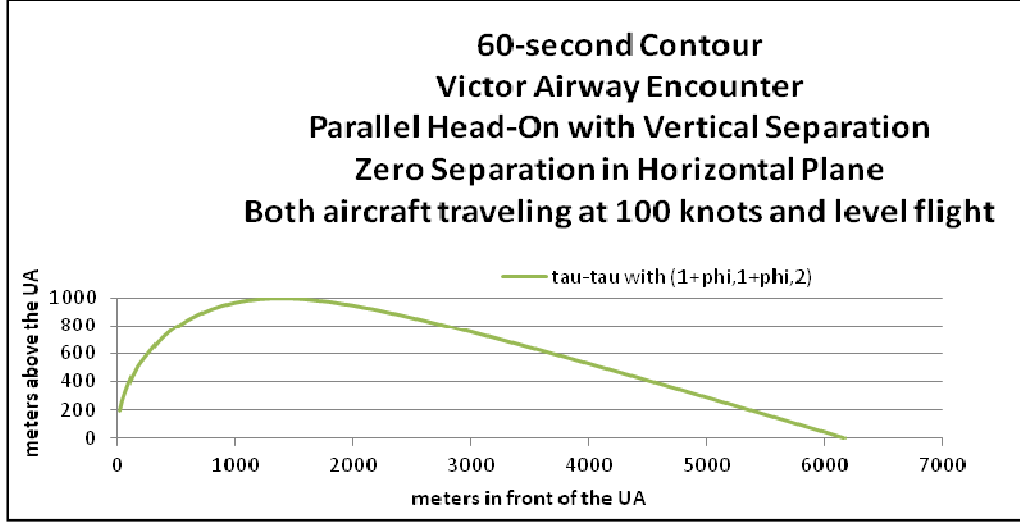


Figure 19. Victor Airway Encounter With UA and the Intruder

The vertical separation increases as the horizontal distance decreases on the contour. Eventually, a maxima is reached in the curve. Depending on the vertical accuracy of the sensors and the vertical maneuverability of the UA, the vertical height of the curve can be scaled down and flattened by the logic designer. This is done by multiplying scaling factor  $k_V$  to the  $(|\varphi_1| + |\varphi_2|)$  term. By requiring  $k_V$  to have the same units as  $\varphi_1$  and  $\varphi_2$ , the time units of the equation are preserved. A good initial scaling factor to consider for  $k_V$  is  $2\pi$  radians. The final tuning of the  $k_V$  term should depend on sensor accuracy and maneuverability.

By adding in horizontal velocity constant  $V_H$  and vertical velocity constant  $V_V$  to account for slow closure rates, Equation 22 becomes:

$$\tau \tau = \frac{(1+k_V|\varphi_1|+k_V|\varphi_2|)(3D \text{ range})}{V_V + V_H + V_{1,z} + (V_1 \cos(\theta_1) + r_1 \omega_1) \cos(\alpha) - V_{2,z} - (V_2 \cos(\theta_2) + r_2 \omega_2) \cos(\beta)} \quad (22)$$

For encounters where there is zero closing velocity in horizontal and vertical planes, the terms  $V_H$  and  $V_V$  create an ellipsoid that is a circle in the x,y plane. This is somewhat similar to the shape of the hockey puck that defines the near mid-air collision volume except that there are no corners.

There is a slight problem with an equation where one aircraft has low velocity (for example, hovering helicopter). If the heading cannot be determined because of uncertainty from low velocity, then  $\varphi$  may begin to vary widely. For low horizontal velocities,  $\varphi$  can be scaled to become less significant. One way to do this is with the scaling factor, as shown in Table 7. This

is one proposed solution and other scaling factors may be more appropriate depending on the sensor performance in terms of vertical uncertainty and update rates.

Table 7. Scaling Factor Applied to  $\phi$  for Low Horizontal Velocities

$ V_1 $ (Knots)	Scaling Factor When $ V_1 $ is in Knots $e^{-(m/(n+p V_1 ))}$ Where $m=1$ , $n=0.1$ , and $p=1$	Scaling Factor When $ V_1 $ is in Knots $e^{-(m/(n+p V_1 ))}$ Where $m=10$ , $n=0.1$ , and $p=1$
0	$4.54 \times 10^{-5}$	$3.7 \times 10^{-44}$
10	0.9057	0.3715
20	0.9514	0.6080
30	0.9673	0.7173
40	0.9753	0.7793
50	0.9802	0.8191
100	0.99006	0.9049
200	0.99501	0.9512

$$\tau = \frac{(1+k_V|\phi_1|e^{-(m/(n+p|V_1|))}+k_V|\phi_2|e^{-(m/(n+p|V_2|))})(3D \text{ range})}{V_V + V_H + V_{1,z} + (V_1 \cos(\theta_1) + r_1 \omega_1) \cos(\alpha) - V_{2,z} - (V_2 \cos(\theta_2) + r_2 \omega_2) \cos(\beta)} \quad (23)$$

#### D. Adding Right-of-Way Rules

With alerting and prioritizing logic, consider that the right-of-way rules are essentially fine tuning the logic. For a co-altitude parallel head-on encounter, the logic may alert slightly sooner depending on whether the intruder aircraft is to the right or to left of the UA trajectory. Inclusion of right-of-way rules can be accomplished by placing an asymmetric factor “ $q$ ,” onto ownship and intruder terms such that when angles  $\alpha$  and  $\beta$  are positive,  $q_1$  and  $q_1$  have a different value then when the angles are negative. For small UAs, the intruder’s ability to perform right-of-way rules may be greatly diminished as a result of an inability to see the UA. Therefore, the ownship and intruder right-of-way rule scaling factors may be different for certain UAs, as shown in Equation 24.

$$\tau = \frac{(1+k_V|\phi_1|e^{-(m/(n+p|V_1|))}+k_V|\phi_2|e^{-(m/(n+p|V_2|))})(3D \text{ range})}{V_V + V_H + V_{1,z} + q_1(V_1 \cos(\theta_1) + r_1 \omega_1) \cos(\alpha) - V_{2,z} - q_2(V_2 \cos(\theta_2) + r_2 \omega_2) \cos(\beta)} \quad (24)$$

#### E. Alerting on Multiple Intruders

When multiple simultaneous intruder aircraft are considered, a SAA maneuver against one aircraft may induce the need to perform a SAA maneuver against another aircraft. To maintain safety, this may necessitate the issuance of alerts (and maneuvers) at an earlier time than if there was a single aircraft. An exception to this might be when the maneuver logic is

intelligent enough to determine that a second maneuver will not be required to avoid the second aircraft.

When there are multiple intruders in a structured and predictable airspace, such as a controlled terminal environment, inverse square criteria might be appropriate for determining the multiple intruder tau-tau time estimates, as shown in Equation 25:

$$\frac{1}{a_1^2} + \frac{1}{a_2^2} + \dots + \frac{1}{a_N^2} = \frac{1}{a_{Total}^2} \quad (25)$$

where  $a_1$  is the tau-tau time of the first aircraft,  $a_2$  is the tau-tau time of the second aircraft, and  $a_{Total}$  is the total tau-tau time for the aggregate of all aircraft. Alerts are issued based on whether the  $a_{Total}$  exceeds some alerting threshold.

When there are multiple intruders in unstructured airspace, such as Class E, inverse criteria may be more appropriate for issuing tau-tau alerts, as shown in Equation 26:

$$\frac{1}{a_1} + \frac{1}{a_2} + \dots + \frac{1}{a_N} = \frac{1}{a_{Total}} \quad (26)$$

where only positive  $a_1, a_2, \dots, a_N$  values are considered. Table 8 shows the inverse and inverse square criteria for determining the multiple intruder time estimates.

Table 8. Alert Combinations Using Inverse and Inverse Square Criteria (Seconds)

					Inverse Criteria Class E Airspace	Inverse Square Criteria Structured Airspace
$a_1$	$a_2$	$a_3$	$a_4$	$a_5$	$a_T$	$a_T$
90					90	90
90	90				45	63.64
90	90	90			30	51.96
90	105	120	135	150	23.23	51.10
90	90	90	90		22.5	45
45	45				22.5	31.82
90	90	90	90	90	18	40.25
45	60	75	90		15.8	30.53
45	60	75	90	105	13.73	29.31
45	45	90	90	90	12.86	27.14

## F. Determining Maneuvers

For completeness of the alerting and prioritization logic, maneuver logic also needs to be considered. For aircraft with low maneuverability, additional scaling of certain tau-tau parameters may be required. If a tau-tau based maneuver logic were developed, it could recommend maneuvers such that  $a_T$  would be maintained above some threshold. When considering maneuvers, the sensor and maneuverability limitations (including wind) should also be well thought-out. If a sensor produces a large amount of track uncertainty in intruder states, maneuvers may need to be started earlier. Likewise, if a UA is not highly maneuverable, UA maneuvers may also need to be initiated earlier. Many UAs are specifically designed to be slow, making wind an important alerting and maneuver parameter for certain systems.

There may not be a single time-based alerting threshold for the initiation of maneuvers for all SAA systems. To tackle this problem, create a time-based threshold that must be maintained. For example, UA maneuvers must be initiated early enough such that, under normal operations, a tau-tau value of less than XX seconds does not occur more than YY percent of the time. Because tau-tau equates a minimum distance to time (assuming zero closing effective rates), distance thresholds are already captured in this approach. Another approach might be to create a target level of safety standard and issue alerts such that risk levels are maintained below some threshold.



## VIII. OTHER TAU-TAU LOGICS

Since the alerting and prioritizing logic is dependent upon sensor inputs, other tau-tau logics exist. Tau-tau logics not only estimate the time to collision, but they prioritize amongst intruders that are not on collision trajectories. Consider the case of an airborne Electro-Optic (EO)/Infrared (IR) camera-based SAA system that measures angles very precisely but does not directly measure range. Such a system may be able to create time estimates to collision without the use of range data or range estimates. This kind of time estimation is found naturally in biological systems and becomes especially useful in navigating various first person video games. In biological systems, a time estimate to collision can be estimated by the angular size of the object divided by the image rate of expansion [3]:

$$\tau_{optical} \approx \frac{\theta}{\frac{\partial \theta}{\partial t}} \approx \frac{\theta \Delta t}{\Delta \theta} \quad [\text{first-order estimate}] \quad (27)$$

where  $\tau_{optical}$  is the first-order optical time estimate to collision, and  $\theta$  is the angular size of the intruder image. The simplified first-order equation predicts the time it will take until the image reaches some angular threshold indicating a collision. Second-order effects can and should also be used. Large accelerating image growth may indicate that collision is imminent and immediate action is required. For the optical tau estimate to be correct, it assumes that the image is on a collision path. To create an optical tau-tau logic, it must also include objects that are not on a collision path and be able to correctly prioritize between them. As such, the angular motion of the intruder image across the field of view becomes an important parameter.

For straight line encounters between objects, those on a collision path will have zero angular rate across the field of view and the image size will grow. For encounters that include constant turn rates, those on a collision path will have decelerating angular rate that reaches zero at the point of a collision. It is important for an EO-based SAA system to include decelerating angular rates in its alerting criteria as many collisions occur when one or more aircraft are maneuvering. It is not enough to only consider the case when angular rates across the field of view are zero. In addition, the angular shape (and angular size) of the object may change during maneuvers. This also has to be carefully considered and accounted for in error analysis.

An optical tau-tau logic could be developed that compares the optical tau estimate against angular motion across the field of view. Table 9 shows the conditions needed for collision and non-collision trajectories. By combining the collision and non-collision parameters, prioritization of intruders that are not on a collision trajectory starts to take shape.

Table 9. Optical Parameter Combinations that Correlate to Collision Risk

<b>Time estimate for when collision occurs with the target image, <math>\tau_{optical}</math></b> <b>Time estimate for when the angular rate of the target image equals zero, <math>\tau_{\frac{\partial\theta}{\partial t}=0}</math></b> <b>Time estimate for when the target image leaves the Field of View, <math>\tau_{FOV}</math></b>		
Does $\tau_{optical}$ equal $\tau_{\frac{\partial\theta}{\partial t}=0}$ ?	Estimated Angular Acceleration of the Target Image Across the Field of View at $\tau_{FOV}$	Outcome
Yes	N/A	Collision
No	N/A	No Collision
No	Large Acceleration	Near Miss
No	Small Acceleration	Well Clear

The development of optical tau-tau criteria could have a great impact on the use of SAA systems for small UAs. To date, the majority of the focus in the SAA community has been on sensors that provide 3-D position of intruder aircraft. Because of size, weight, and power constraints, passive optical sensors are a desired sensor for small UAs performing SAA against non-cooperative intruder aircraft.

## IX. ALTERNATIVES TO TIME-BASED LOGIC AND GROUND-BASED SENSE AND AVOID FIELDING PLANS

Alternatives to time-based alerting and prioritization logic for SAA exist. An example is the work being done by the Massachusetts Institute of Technology (MIT) Lincoln Laboratory on their ACAS (Airborne Collision Avoidance System) logic. MIT Lincoln Laboratory is building the next generation of collision avoidance logic for the Federal Aviation Administration (FAA) [4], as well as SAA logic for the Army. The development incorporates dynamic programming to create an optimized logic table. The maneuver logic uses states (which include sensor measurements of position and velocity) as inputs to determine optimally safe maneuvers. The logic does not necessarily need to use a time estimate to collision as an input nor does it necessarily need to internally calculate a time estimate to collision to determine safe maneuvers. Because nuisance alerts are a consideration in the dynamic programming, false alarms are minimized. A product of having a self-consistent alerting and maneuver logic that minimizes nuisance alarms is that the logic naturally prioritizes very well. The logic exhibits all of the desired attributes discussed in this report, with the exception of providing a time estimate since it is not needed for maneuver selection within the logic.

The current plan within the Army's Unmanned Systems Airspace Integration Concepts Product Office is to utilize ACAS-based logic, which has been developed specifically for Ground-Based SAA (GBSAA), and a time estimation logic for situational awareness. The time estimation logic will provide self-separation traffic advisories at time scales larger than required to maintain self-separation. If a scenario progresses and self-separation is required, then the GBSAA ACAS logic will escalate the alert and self-separation maneuvers will be performed.

Redundant, diverse, and layered alerting logic provides greater safety during the initial fielding of GBSAA systems.

The current schedule does not allow for the investigation and optimization of a tau-tau logic for self-separation traffic advisories during the initial fielding of GBSAA systems. Therefore, there may be a higher number of tau-based nuisance alerts for self-separation traffic advisories than in the planned follow-on upgrades. Operator requirements for initial systems will be set higher than for follow-on upgrades. It is expected that operator requirements will diminish as the system matures with each new advancement in the technology.

## **X. SUMMARY**

Prioritization of multiple intruders is important for human interactions with a SAA system. If time-based alerting and prioritizing logic is used, it should be well integrated for correct prioritization and to reduce nuisance alerts. Time-based logic should have in the assumptions collision and non-collision encounters. As a result of the great variability between UA maneuverability and sensor types, there may be specific tuning for each SAA system (or class of SAA systems) to support a specific UA (or class of UA) and operational capability.

A theoretical tau-tau alerting and prioritizing logic has been outlined in detail for SAA systems employing 3-D sensors. This logic appears to be potentially superior to tau-based logics that have been used in the past. Before implementing the tau-tau concept, additional work is needed to verify the performance and behavior. Logic tuning may be required for a specific SAA implementation.

A conceptual construct of an optical tau-tau alerting and prioritizing logic for EO/IR camera systems has also been proposed. More work is needed in this area. The introduction of small Unmanned Aircraft Systems (UASs) into the National Airspace System may lead to a greater market share than held by large UASs. The SAA and UAS certification standards for small UAS operations beyond line of sight would enable a revolutionary capability that would have utility across a multitude of disciplines and applications.

## REFERENCES

1. Introduction to TCAS II Version 7.1, U.S. Department of Transportation, Federal Aviation Administration, 28 February 2011.
2. Kuchar, J. K., “Evaluation of Proposed Changes to the ACAS Modified Tau Calculation,” ICAO SCRSP WG A/WP A10-03, 2006, retrieved 28 November 2012, [http://www.ll.mit.edu/mission/aviation/publications/publication-files/ms-papers/Kuchar\\_2006\\_SCRSP\\_MS-24330\\_WW-18698.pdf](http://www.ll.mit.edu/mission/aviation/publications/publication-files/ms-papers/Kuchar_2006_SCRSP_MS-24330_WW-18698.pdf)
3. Yan, Jing-Jiang et al., “Visual Processing of the Impending Collision of a Looming Object: Time to Collision Revisited,” Journal of Vision, Volume 11, Number 12, Article 7, 13 October 2011, retrieved in 05 February 2013, <http://www.journalofvision.org/content/11/12/7.full>
4. Kochenderfer, M.J.; Holland, J.E.; and Chryssanthacopoulos, J.P.; “Next-Generation Airborne Collision Avoidance System,” Lincoln Laboratory Journal, Volume 19, Number 1, 2012, retrieved 1 February 2013, [http://www.ll.mit.edu/publications/journal/pdf/vol19\\_no1/19\\_1\\_1\\_Kochenderfer.pdf](http://www.ll.mit.edu/publications/journal/pdf/vol19_no1/19_1_1_Kochenderfer.pdf)

## **LIST OF ABBREVIATIONS, ACRONYMS, AND SYMBOLS**

%	percent
°	degree
3-D	Three-Dimensional
ABSAA	Airborne-Based SAA
ACAS	Airborne Collision Avoidance System
CPA	Closest Point of Approach
DMOD	Distance Modification
EO	Electro-Optic
FAA	Federal Aviation Administration
fpm	feet per minute
GBSAA	Ground-Based SAA
IR	Infrared
kts	knots
MIT	Massachusetts Institute of Technology
N/A	Non-Applicable
NM	Nautical Mile
NMAC	Near Mid-Air Collision
R of W	Right-of-Way
RA	Resolution Advisory
SAA	Sense and Avoid
SL	Sensitivity Level
SWAP	Size, Weight, and Power
TA	Traffic Advisory
TCAS	Traffic and Collision Avoidance System
UA	Unmanned Aircraft
UAS	Unmanned Aircraft System

## INITIAL DISTRIBUTION LIST

		<u>Copies</u>
Weapon Systems Technology Information Analysis Center Alion Science and Technology 201 Mill Street Rome, NY 13440	Ms. Gina Nash <a href="mailto:gnash@alionscience.com">gnash@alionscience.com</a>	Electronic
Defense Technical Information Center 8725 John J. Kingman Rd., Suite 0944 Fort Belvoir, VA 22060-6218	Mr. Jack L. Rike <a href="mailto:jrike@dtic.mil">jrike@dtic.mil</a>	Electronic
AMSAM-L	Ms. Anne C. Lanteigne <a href="mailto:hay.k.lanteigne.civ@mail.mil">hay.k.lanteigne.civ@mail.mil</a> Mr. Michael K. Gray <a href="mailto:michael.k.gray7.civ@mail.mil">michael.k.gray7.civ@mail.mil</a>	Electronic Electronic
RDMR		Electronic
RDMR-CSI		Electronic
RDMR-TM	Mr. Adam G. Hendrickson <a href="mailto:adam.g.hendrickson.civ@mail.mil">adam.g.hendrickson.civ@mail.mil</a>	Electronic/Hardcopy